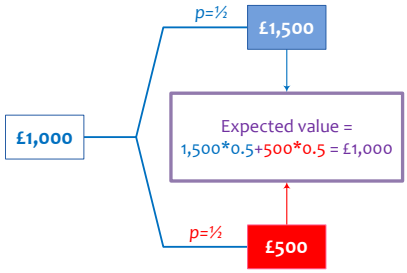
investanalitix
finance technology | investment intelligence

IX University Series

Understanding the Construction and Applications of an Index of Risk Aversion

Understanding the Construction and Applications of an Index of Risk Aversion

Consider a hypothetical *fair game* in which you are presented with the following two options: Take a guaranteed £1,000 and walk away or participate in a speculation in which you toss a fair coin and win or lose, with equal certainty, 50% of the guaranteed amount. To sum up, a tail will get you total winnings of £1,500 but a head will see you take home only £500. Theoretically, you figure, the value of both outcomes is £1,000, so which of these options would you choose?



It is reassuring to know that this is not a trick question and there is no right or wrong answer! The reply, therefore, is subjective and your choice dependent on your *attitude toward risk* or, in the parlance of behavioural finance, your *cognitive bias*. If you reject this game, then you are a risk-averse investor. If you reject the guaranteed sum in favour of the speculation, you are a risk-lover. If you are indifferent to both, you are known as a risk-neutral investor: you would take the same view of either option.

Concept Check

A fair game is a prospect with zero risk premium. In a fair game, the expected value of the uncertain sum is equal to that of the certain sum.

Now let's tweak the terms of the game and create a slight but important paradigm shift. Suppose that you have to invest £1,000 of *your own* funds and that this is your entire wealth. All the other parameters of the game are unchanged. For simplicity, we'll also assume that you are a risk-averse investor. The question we now pose for you is this:

"What is the maximum amount you would be willing to speculate?"

Suppose you say that you are only willing to speculate 25% of your total wealth on the risky prospect. If you are successful, you stand to increase your wealth by 12.5%, thus: £1,000 + (£250*0.5) = £1,125. If you are unsuccessful, your wealth will diminish by 12.5%: £1,000 - (£250*0.5) = £875.

What does your choice say about your risk profile and investment preferences? Firstly, it gives an indication of your risk tolerance by quantifying it: £250/£1,000 = 0.25 – we have just defined a *coefficient of relative risk tolerance* for you! Second, it tells us something about your *degree* of risk aversion: If we take the inverse of your coefficient of risk tolerance, we have your *index of risk aversion*. Table 1 is a summary of your choices and what they imply:

Table 1: Constructing a Simple Index of Risk Aversion

Your wealth	Sum invested in the risky prospect	Coefficient of risk tolerance	Index of risk aversion
£1,000	£250	£250/£1,000 = 0.25	1/0.25 = 4

Your index of risk aversion is 4, which places your profile on the more conservative side of the "representative" investor whose index is typically in the range of 2-4. The full spectrum for a risk-averse investor is between 1 and 7; 1 signifying least risk-averse and 7, most risk-averse.

While we have greatly simplified the process, the industry methodology for assessing an investor's risk profile is essentially as we have described it through our game above. In practice, advisors routinely evaluate an investor's risk aversion through qualitative interviews combined with quantitative questionnaires (of varying quality) which pit risky prospects against riskless ones to "test" each candidate's aptitude for risk under various scenarios. This psychological profiling is often quite accurate and yields a reasonably reliable picture of an investor's attitude toward risk. However, there is no standardization and the results of one questionnaire are not necessarily the same as another's. It is therefore useful for advisors to "supplement" such questionnaires by engaging with investors further through educational games, samples of which are provided in this article.

Table 2: Portfolio Options

	σ_R	E_R
Investment	Risk (Volatility %)	Expected return (%)
1	0	10.0
2	5	10.5
3	10	12.0
4	15	14.5
5	20	18.0
6	25	22.5
7	30	28.0
8	35	34.5
9	40	42.0
10	45	50.5

Now let's move on to the practical world and look at typical scenarios you might encounter when making everyday investment decisions. As an investor with a risk aversion index of 4, you are presented with the choice of 10 portfolios with the risk/return characteristics outlined in Table 1. Ponder over the figures a while; which investment will you choose?

Concept Check

A utility value is the welfare assigned by an investor to a given investment prospect based on his/her assessment of its risk/reward characteristics

Before you fret over the question too much, it's only fair to say that this one *IS* a trick question! To see the trick, we need to calculate the *utility value* of each of these options. This is the unique value that you put on each prospect based solely on your risk profile and attitude. In other words, you propose to answer the very personal question: *"What is this investment worth to me?"*

To answer this question, we need to introduce a formula that encodes and links the three key factors discussed above, that is: The expected return of each investment; the risk of each investment; and your index of risk aversion. That formula is:

$$U_R = E_R - \tfrac{1}{2}A(\sigma_R)^2$$

The formula essentially says that in order to arrive at your unique utility value (U_R) for each portfolio, you adjust its expected return (E_R) by deducting from it a risk penalty ($- A\sigma_R^2$). This penalty is commensurate with your degree of risk aversion, as indicated by your index of risk aversion (A). In Table 3, we have used your risk aversion index of 4 to calculate the U_R value for each of the risk and return figures provided for the ten portfolios.

Table 3: Utility of Portfolio Options

	σ_R	E_R	U_R
Investment	Risk (Volatility %)	Expected return (%)	Expected utility (%)
1	0	10.0	10.0
2	5	10.5	10.0
3	10	12.0	10.0
4	15	14.5	10.0
5	20	18.0	10.0
6	25	22.5	10.0
7	30	28.0	10.0
8	35	34.5	10.0
9	40	42.0	10.0
10	45	50.5	10.0

Notice anything unusual? You have spotted it – the numbers are identical! For you, the expected utility of each investment is equal to that of option 1, the risk-less investment (volatility = 0%). Therefore, you see each of these investments as being the same in terms of the risk-adjusted value they offer – their utility value to you is 10% across the board. In other words, you are *indifferent* to selecting among them and are just as likely to choose the investment with a "guaranteed" expected return of 10% (option 1) as the investment with an expected return of 50.5% but volatility of 45%. Since the expected utility of all these permutations of risk and return is equal to that of the risk-less investment (the one whose outcome is *certain*), we can say that each of these investments provides an expected return which is the *certainty equivalent* of the risk-less rate.

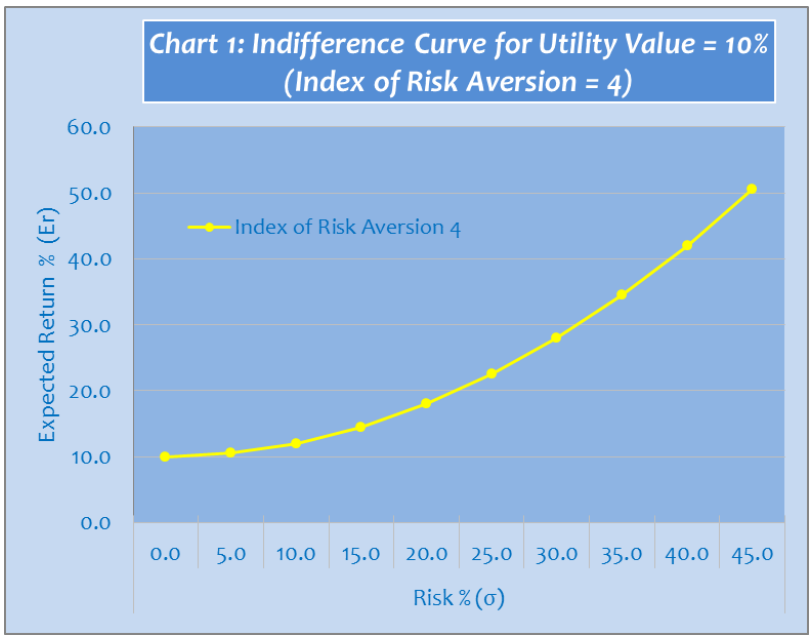
Concept Check

Certainty equivalent is the certain return that provides the same level of utility as the risky prospect.

If we plot all the utility values in Table 3 on a graph they will trace an *indifference curve* – you will be indifferent among investments represented by *any* risk/return points that sit on this curve, all of which yield a utility of 10%. We have constructed such a curve in Chart 1. Notice its shape: It is convex and shows that, as risk increases, you demand a commensurate increase in expected return for the portfolio in question. Furthermore, the curve progressively steepens as risk aversion increases and flattens as it is diminishes, as we shall see shortly.

Concept Check

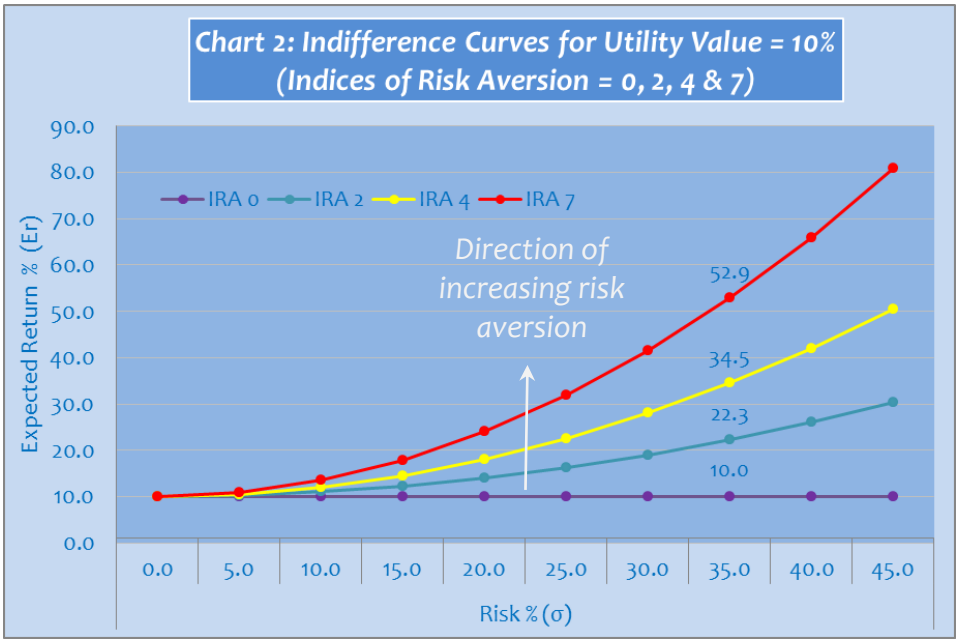
An indifference curve is a curve that connects all investments that yield the same level of utility for all given risk and return values



To put your level of risk aversion in context, Chart 2 shows the indifference curves for the same level of utility (U_R=10%) of two other investors, one more risk-averse than you (IRA=7) and the other less so (IRA=2).

The first point to notice in Chart 2 is the steepening/flattening of the indifference curve as risk aversion increases/decreases. Thus, the investor with a risk aversion index of 7 has an indifference curve which is noticeably steeper than yours. In turn, your curve is steeper than that of the investor with an index of risk aversion of 2.

The second point of interest is how perceptions of the *same* portfolio vary across the spectrum of risk aversion. While all three risk-averse investors see a utility value of 10% across all ten portfolios, when adjusted for their level of risk aversion, the required expected return for yielding this level of utility is significantly different at each risk point.



Using portfolio #8 (E_R=34.5% and σ_R=35%) as a reference point, the chart illustrates each investor's perspective on its risk/reward profile. For you, for example, an expected return of 34.5% is sufficient compensation for the portfolio's 35% volatility. For the more risk-averse investor (IRA=7), however, this figure is significantly higher at 52.9% - he has a more cautious view of that level of volatility and requires a much higher expected return before he ascribes the same utility value (10.0%) to the investment. On the other hand, the less-risk-averse investor is satisfied with an expected return of only 22.3% - he takes a much-less punitive view of the same level of volatility.

What of the last "curve"? This corresponds to the risk-neutral investor. As the name implies, for him risk is irrelevant and regardless of the level of volatility, his perception of the utility value of each of the portfolios is the same.